Mathematics Workbook

How to use SET® in the classroom

TABLE OF CONTENTS

INTRODUCTION

The Mathematics Workbook is a collection of tips and techniques from teachers, doctors of philosophy, and professors, on how to use the SET® Game in the classroom. SET has been used in schools throughout the United States and Canada in grades K-12 and university level classes to enhance **Cognitive, Logical and Spatial Reasoning, Visual Perception, Math Skills, Social Skills and Personal Skills**.

This workbook is intended to provide guidance on how to integrate SET into your current curriculum. As we receive additional materials from teachers around the country, we hope to continually expand the content of the workbook. If you have constructed exercises using the SET game as part of your curriculum and would like to have them considered for the workbook, please send them to the address below. If your material is accepted, you will receive full acknowledgement as well as a free game.

Mail:
Set Enterprises, Inc.
16537 E. Laser Drive, Suite 10
Fountain Hills, AZ 85268

Fax:
480-837-5644

Email:
setgame@setgame.com

If you have questions, please call us at 1-800-351-7765.

SET'S SKILL CONNECTIONS

SET
The Family Game of Visual Perception®

Teaches: **Cognitive, Logical and Spatial Reasoning, Visual Perception, Math Skills, Social Skills and Personal Skills**

Ages: 6 and up
Players: 1-20

SET'S Skill Connections

SET is the award-winning puzzle game that is truly a challenge for the whole class. Students in grades kindergarten through college can challenge one another – age and experience are not advantages because SET draws on fundamental thinking processes. Play is simple. To start, deal out 9 cards from the deck of solid shaded symbols (the smaller deck in the box), arranging them on a table in a three by three array. The game has only one rule: find three cards that are all the same or all different in each of their three attributes, when looked at individually. The features are ***symbols, color***, and the ***number*** of symbols. There are no turns, everybody is playing at once. The first student to see a *SET*, shouts "*SET*", everybody else freezes for a few seconds to let that student pick up the *SET*. Each *SET* is worth one point. The dealer replaces the three cards and the play continues. The game is fast and furious. The student with the most *SETs* at the end wins.

SET builds **cognitive, logical and spatial reasoning skills** as well as **visual perception skills**. Because it has a rule of logic (three cards that are all the same *or* all different in each individual attribute), and because students must apply this rule to the spatial array of patterns taken all at once, they must use both **left brain and right brain thinking skills**. When players can easily find *SETs* using only the solid cards, add the rest of the cards that are in the package. This brings the deck up to 81 cards and adds the attribute of ***shading*** to the list of attributes that must satisfy the rule to be a *SET*. Playing with all four attributes exponentially increases the difficultly level of finding a *SET*. Now, deal out 12 cards from the entire deck of 81 cards in a 3 x 4 array. The play is the same as before, except now a *SET* must be either all the same or all different in all four attributes, when looked at individually, across all three cards. Understanding and identifying *SETs* is, of course, an important **math skill**.

Social skills develop naturally as students play in groups and follow the rules of the game. Any number of people can play at one time. **Personal skills** are enhanced as self-esteem grows with the finding of *SETs*. There is no luck in the game, so every *SET* found is a personal victory whether playing solitaire or in a crowd. Furthermore, it is a challenge that never gets old, since every time three cards are laid down to replace a *SET*, the challenge starts anew.

Best Game Awards & Recognitions

SET® has won the following 38 Best Game Awards & Recognitions

MENSA Select Award	1991
The Detroit News	1991
OMNI Magazine	1991
The Consumers Association of Quebec	1992
The Canadian Toy Testing Council	1992
Games Magazine Games 100 Award	1992
Games Magazine Games 100 Award	1993
Games Magazine Games 100 Award	1994
Games Magazine Games 100 Award	1995
Dr. Toy's 10 Best Games	1996
Dr. Toy's 100 Best Children's Products	1996
ASTRA Top Toy Pick	1996
Parents' Choice Award	1997
Parents Magazine	1998
Parents' Council Award	1999
Top Ten Games – Wizards of the Coast	2000
Teachers Choice Learning Award	2001
Educational Clearinghouse A+ Award	2001
Bernie's Major Fun Award	2002
NSSEA – Top New Product	2002
ASTRA Hot Toys	2004
Parents' Choice Best 25 games of the past 25 years	2004
Top 100 Games of 2005 Games Quarterly	2005
TD*monthly* Top-10 Most Wanted Card Games	2006
TD*monthly* Top-10 Most Wanted Card Games	2007
Creative Child's Preferred Choice Award	2007
TD*monthly* Classic Toy Award	2007
TD*monthly* Top-10 Most Wanted Card Games	2008
TD*monthly* Top-10 Most Wanted Card Games	2009
TD*monthly* Top-10 Most Wanted Card Games	2010
TD*monthly* Top Seller	2010
NAPPA Children's Products Honors Winner	2010
TD*monthly* Top-10 Most Wanted Card Games	2011
TD*monthly* Top-10 Most Wanted Card Games	2012
Sharp As A Tack Outstanding Educational Value – Cognitive Processing Speed	2013
ASTRA Best Toy for Kids Award - Classic Toy Finalist	2013
TD*monthly* Top-10 Most Wanted Card Games	2013
TD*monthly* Top-10 Most Wanted Card Games	2014

HOW TO PLAY SET®

Rules

The object of the game is to identify a *SET* of three cards from 12 cards laid out on the table. Each card has a variation of the following four features:

(A) Color:
Each card is *red*, *green*, or *purple*.

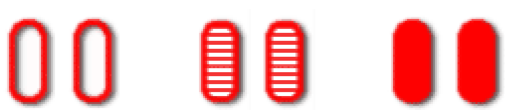
(B) Symbol:
Each card contains *ovals*, *squiggles*, or *diamonds*.

(C) Number:
Each card has *one*, *two*, or *three* symbols.

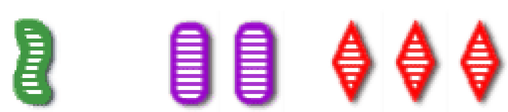
(D) Shading: Each card is *solid*, *open*, or *striped*.

A *SET* consists of three cards in which each individual feature is EITHER the same on each card OR is different on each card. That is to say, any feature in the *SET* of three cards is either common to all three cards or is different on each card.

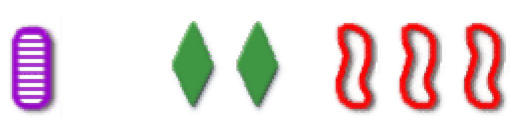
For example, the following are *SETs*:



*All three cards are **red**; all are **ovals**; all have **two symbols**; and all have different **shadings**.*

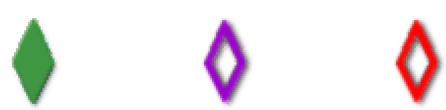


*All have different **colors**; all have different **symbols**; all have different **numbers of symbols**; and all have the same **shading**.*

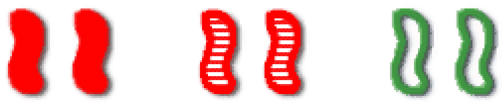


*All have different **colors**; all have different **symbols**; all have different **numbers of symbols**, and all have different **shadings**.*

The following are <u>not</u> *SETs*:



*All have different **colors**; all are **diamonds**; all have **one symbol**; however, two are **open and one is not.***



All are ***squiggles****; all have different* ***shadings****; all have* ***two symbols****; however, two are* ***red*** *and one is not.*

The Magic Rule

If two are... and one is not, then it is not a *SET*.

Quick Start

For a quick introduction for anyone playing the card version, and especially for children under six, start with the small deck (*just the solid symbols*). This eliminates one feature, shading. Play as indicated below but only lay out nine cards. When you can quickly see a *SET* with this 27 card mini version, shuffle the two decks together.

The Play

The dealer shuffles the cards and places twelve cards (*in a rectangle*) face up on the table so that they can be seen by all players. The players remove a *SET* of three cards as they are seen. Each *SET* is checked by the other players. If correct, the *SET* is kept by the player and the dealer replaces the three cards with three from the deck. Players do not take turns but pick up *SETs* as soon as they see them. A player must call *SET* before picking up the cards. After a player has called *SET*, no other player can pick up cards until the first player is finished. If a player calls *SET* and does not have one, the player loses one point. The three cards are returned to the table.

If all players agree that there is no *SET* in the twelve cards showing, three more cards (*making a total of fifteen*) are placed face up. These cards are not replaced when the next *SET* is picked up, reducing the number to twelve again. If solitaire is being played, the player loses at this point.

The play continues until the deck is depleted. At the end of the play there may be six or nine cards which do not form a *SET*.

The number of *SETs* held by each player is then counted; one point is given for each and added to their score. The deal then passes to the person on the dealer's left and the play resumes with the deck being reshuffled. When all players have dealt, the game ends; the highest score wins.

SET Interactive Tutorial

The SET interactive tutorial is available on our website at the link below. In the SET tutorial, you'll meet your interactive guide, "Guy". Guy is there to walk you through how to play SET and show you how to make a *SET*.
http://www.setgame.com/sites/default/files/Tutorials/tutorial/SetTutorial.swf

QUICK INTRODUCTION TO SET®

HOW TO PLAY SET®

For a quick introduction to the SET game, play using *only the SOLID* shaded cards. This makes it *<u>much easier</u>* to learn what a *SET* is and how to play.

To find a SET you must answer the first THREE questions with a "YES".

1. Is the **COLOR** on EACH card either all the same or all different?
2. Is the **SYMBOL** on EACH card either all the same or all different?
3. Is the **NUMBER** of symbols on EACH card either all the same or all different?

When this becomes easy for you, play with the <u>full</u> deck and answer all FOUR questions with a "YES".

4. Is the **SHADING** (solid, outlined, or shaded) on EACH card either all the same or all different?

-- ✂ --

HOW TO PLAY SET®

For a quick introduction to the SET game, play *using only the SOLID* shaded cards. This makes it *<u>much easier</u>* to learn - what a *SET* is and how to play.

To find a SET you must answer the first THREE questions with a "YES".

1. Is the **COLOR** on EACH card either all the same or all different?
2. Is the **SYMBOL** on EACH card either all the same or all different?
3. Is the **NUMBER** of symbols on EACH card either all the same or all different?

When this becomes easy for you, play with the <u>full</u> deck and answer all FOUR questions with a "YES".

4. Is the **SHADING** (solid, outlined, or shaded) on EACH card either all the same or all different?

MATHEMATICAL FUN & CHALLENGES IN THE GAME OF SET®

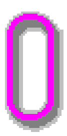
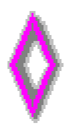
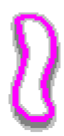
By Phyllis Chinn, Ph.D. ***Professor of Mathematics***
Dale Oliver, Ph.D. ***Professor of Mathematics***
Department of Mathematics
Humboldt State University
Arcata, CA 95521

The Game of SET

In 1988 Marsha Falco copyrighted a new game called SET. This game proves to be an excellent extension for activities involving organizing objects by attribute. In addition to reinforcing the ideas of sameness and distinctness, the SET game, and variations on it, provide an interesting and challenging context for exploring ideas in discrete mathematics. Even though the NCTM's 1989 *Curriculum and Evaluation Standards for School Mathematics* includes discrete mathematics as a standard for grades 9-12, the activities suggested here are strongly supported by the K-4 and 5-8 standards involving mathematics as problem solving, communication, and reasoning.

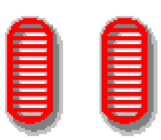
The SET deck

The game of SET is a card game. A single card is identified by four attributes: number, shape, color, and shading. The full deck of cards form a complete set of all possible combinations of the four attributes. Each card has one, two or three (number) copies of the same figure showing. The figures are one of three shapes, colored with one of three colors, and shaded in one of three ways. In the commercial game, the shapes are

  AND 

called "oval", "diamond" and "squiggle" respectively. Each of these shapes may be colored purple, red or green, and each is either outlined, filled in or striped. For example, the card in figure 1 has number 2, shape oval, color red, and shading striped. No two cards in the deck are identical and each possible choice of one value for each attribute occurs on one card.

Figure 1



When introducing SET in your classroom, challenge your students to describe the full deck of SET cards for themselves. Include in this challenge the question "Can you determine without counting the cards one by one, how many cards are in the complete SET deck?" Let the students have a deck to work with and ask them to figure out the rule by which the deck was constructed, or have the students construct a deck themselves and figure out in advance how many cards they will need. There are many ways children might arrive at the full count, usually involving some sorting of the cards.

The process of counting the SET deck cards without counting the cards one by one illustrates one the basic counting principles of discrete mathematics, called the multiplication principle. This principle says "if a first event can occur in n ways, and for each of these n ways a second event can occur in m ways, then the two events can occur in m x n ways. Here the "events" are the number of ways to assign attributes to the SET cards. For any card, one can choose 3 different number of figures to display, combined with one of three shapes for 9 combinations. Each of these 9 combinations can be paired with one of 3 colorings in 9 x 3 = 27 ways, each of which can be paired with 3 shadings for a total of 27 x 3 = 81 cards in the deck.

A 'Set' of Three

Sets of three cards from the SET deck which satisfy the condition that all the cards either agree with each other or disagree with each other on each of the four attributes (number, shape, color, and shading) are the fundamental objects in the SET game. Three cards form a 'set' if the cards display the same number of figures or each display a different number of figures, AND if the figures are all the same shape or three different shapes, AND if the figures are the same color or three different colors, AND if the figures are shaded with the same shading or three different shadings. For example, the cards in Figure 2 are a 'set', but the cards in Figure 3 are not a 'set'. Can you tell why?

Figure 2

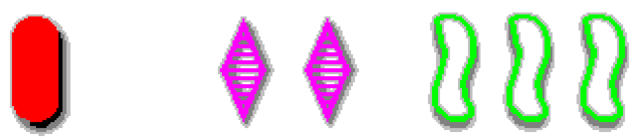
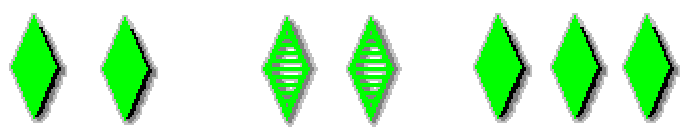


Figure 3

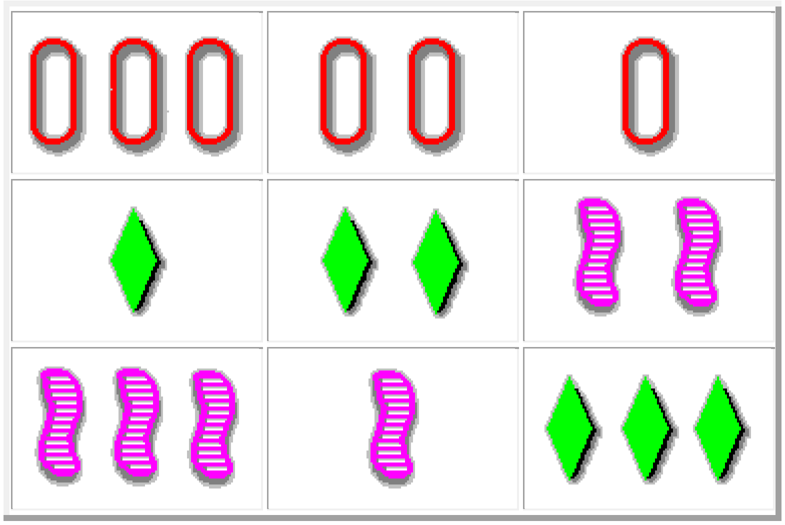


Playing SET

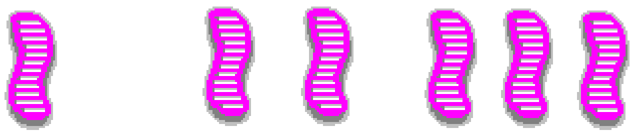
To begin the game of SET, the dealer shuffles the cards and lays some of them out in a rectangular array. (The official rules suggest beginning with 12 cards. From an educational point of view, it may be simpler for children to play beginning with 9 cards.) All players look at the same layout of cards seeking a 'set' of 3 cards as defined above. According to the official rules, there is a "MAGIC" rule: if two cards are.....and one is not ...., then it is not a 'set'."

To practice your understanding of the definition, see how many 'sets' you can find in Figure 4.

Figure 4



Did you find the 'set' consisting of:



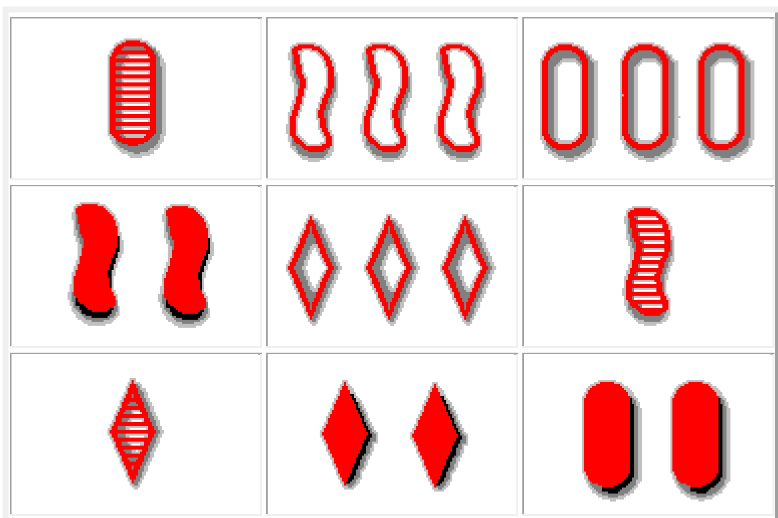
How about the 'set' consisting of:

Notice that the three cards in a 'set' may be different in 1,2,3, or 4 of the attributes. The first person to notice a 'set' in the current layout calls out the word 'set' and then is allowed to touch the three cards. While it is not required in the rules, from a pedagogical point of view it is a good idea for the student to explain how s/he knows it is a 'set' -- for example the first 'set' above would be explained by saying, "they are all purple, all striped, all squiggles, and there is a 1, a 2, and a 3 of them." Assuming the student has correctly identified a 'set' s/he takes the 3 cards. If there are now fewer cards in the layout than at the start (i.e., 12 or 9), the dealer replaces them with three new cards. If all players agree there are no 'sets' in the layout, then 3 more cards are added. Play ends when no new cards are left in the deck and no 'sets' remain in the final array. The official game rules suggest that each player keep his/her own score by counting 1 point for each correctly identified 'set', and a -1 point for each incorrect attempt to identify a 'set'. The winner of the game is the player with the most points after each player has had a turn to deal the entire deck. When using SET in the classroom, we suggest a modification of the official rules. For beginners, don't exact any penalty for an incorrect attempt to identify a 'set'. Once students understand the game thoroughly, any student who makes an incorrect attempt may be penalized by not being allowed to call 'set" again until someone else has found a 'set'.

SET and Discrete Mathematics

As mentioned earlier, SET involves discrete math. According to John A. Dossey, "Discrete mathematics problems can be classified in three broad categories. The first category, *existence problems*, deals with whether a given problem has a solution or not. The second category, *counting problems*, investigates how many solutions may exist for problems with known solutions. A third category, *optimization problems*, focuses on finding a best solution to a particular problem."[1] The game of SET presents problems in both of the first two categories. One existence problem is to have each student pick out a random two cards from the SET deck and figure out how many, if any, cards can be found in the deck which can be paired with the first two cards to complete a set. It may take several selections of pairs of cards for students to realize that any pair can be completed to a 'set' by exactly one third card. Once students realize this, encourage them to explain to one another how they can be sure. The result holds for any pair of cards. A sample of such an argument might state: the unique third card is defined attribute by attribute -- for each attribute where the two chosen cards are alike, the third one has the same value; if they are different, the third one has the missing valve. Since only one card has each particular selection of four values for the four attributes, there is a unique completion for a 'set'. This activity supports an atmosphere of mathematics as communication and reasoning in your classroom. Those students who have had more experience counting combinations and permutations can be asked a more challenging question: if you pick any one card from the deck to how many distinct 'sets' does it belong? The answer requires the preceding result, namely that any two cards belong to exactly one 'set'. A particular card forms a 'set' with any of the 80 other cards in the deck with a unique third card to complete that 'set'. Each 'set' with the same beginning card is counted twice -- once with each of the other cards in the 'set' as the 'second' card selected. Thus, there are 80 /2 = 40 'sets' containing the first card.

Figure 5



Another question that junior high students might be able to answer is, "What is the largest number of 'sets' that can be present among a layout of nine cards?" A similar argument to the preceding one suggests that there are 9 possible first cards, each paired with 8 possible second cards -- but any of these cards in a particular 'set' can be 'first' and either of the remaining two can be "second" -- so there are (9x8)/(3x2) = 12 'sets' possible. The layout of Figure 5 is one example of nine cards (all of one color) including 12 'sets'. Can you find them all? Have your students construct their own examples of such layouts. See who can find a layout of 12 cards with the greatest number of 'sets'. Hint 14 is best possible.

Each of the suggested questions may be extended by varying the number of attributes or the number of options for attributes. What about a three-attribute deck with 5 possibilities for each attribute? There would be 125 cards in the deck, with a 'set' defined for a set of 5 cards.

There are many other games that can be played with the SET deck. The game and rules for variations can be obtained from Set Enterprises, Inc. 16537 E. Laser Dr., Ste. 6, Fountain Hills, AZ 85268. Other variations include the games that can be played with other sets of attribute blocks. For a book with many good ideas of attribute activities see [2].

As a final suggestion, 'set' is a word with meanings that are easily confused with the particular triples of the game SET. It might be better for children to call out some other word -- like 'triple' or three or '3-set' or some other word the class selects to describe the particular 'set' for this game.

Despite these minor concerns, the authors think the game of SET is a wonderful activity to add to the classroom -- it is thought provoking and fun!

References

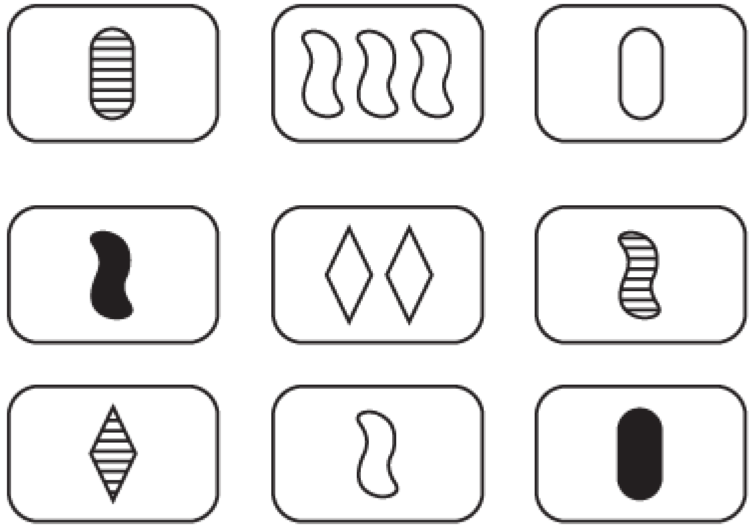
[1] Dossey, John A., "Discrete Mathematics: The Math for our Time",
Discrete Mathematics Across the Curriculum K-12, 1991, NCTM, pp.1-2.

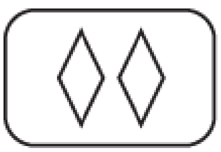
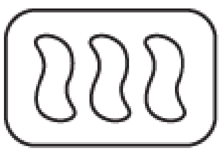
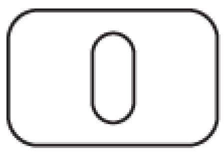
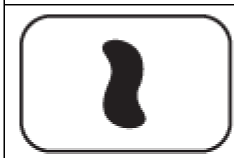
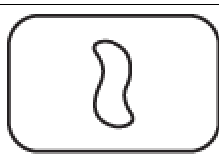
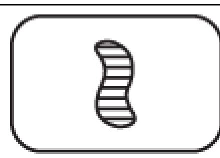
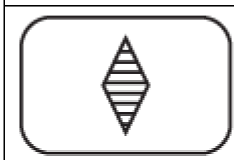
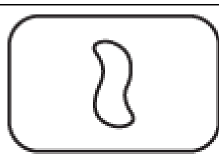
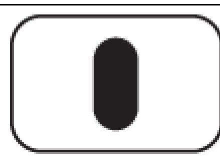
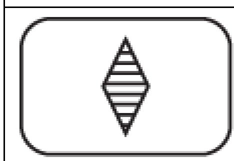
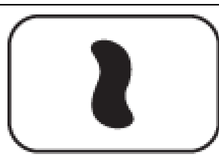
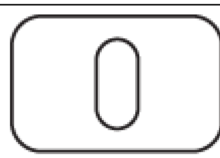
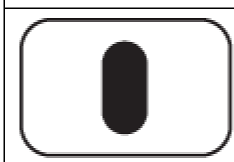
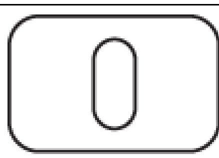
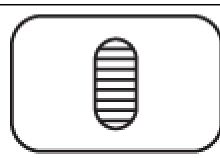
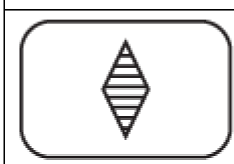
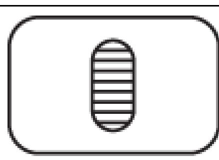
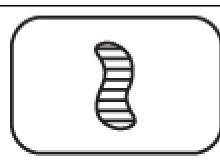
[2] **Teacher's Guide for Attribute Games and Problems**, Elementary Science Study,
Webster Division, McGraw-Hill Book Company, 1968, Educ. Dev. Ctr., Public Domain after 1971.

[3] **Curriculum and Evaluation Standards for School Mathematics**, NCTM, 1989.

CLASSROOM EXERCISES USING SET®

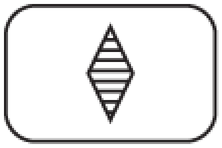
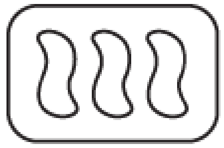
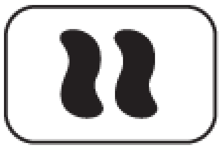
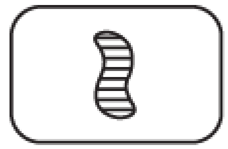
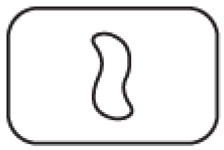
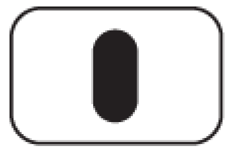
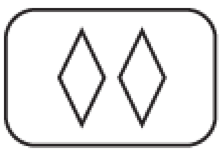
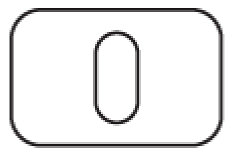
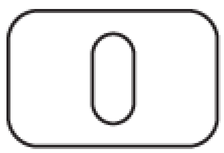
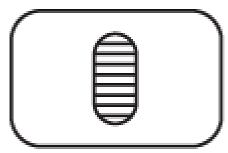
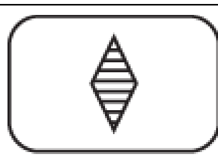
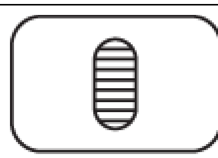
Find the six *SETs* of three cards in the layout below.



The answers are			Because
			The symbols are **different** on each card. The shading is the **same** on each card. The number of symbols is **different** on each card.
			The symbols are the **same** on each card. The shading is **different** on each card. The number of symbols is the **same** on each card.
			The symbols are **different** on each card. The shading is **different** on each card. The number of symbols is the **same** on each card.
			The symbols are **different** on each card. The shading is **different** on each card. The number of symbols is the **same** on each card.
			The symbols are the **same** on each card. The shading is **different** on each card. The number of symbols is the **same** on each card.
			The symbols are **different** on each card. The shading is the **same** on each card. The number of symbols is the **same** on each card.

Draw the Missing Card

For any two cards, there is one unique card that will complete the *SET.*

1)			
2)			
3)			
4)			
5)			
6)			

1. Two solid ovals
2. Three outlined squiggles
3. One striped diamond
4. Three outlined squiggles
5. One solid oval
6. One striped squiggle

SET THEORY USING THE GAME SET®

By Professor Anthony Macula
Michael J. Doughtry
State University of New York at Geneseo
Geneseo, New York 14454

The game SET is an excellent way to introduce basic set theory. It provides a concrete model for understanding and a tool for working through set operations. Students should be encouraged to use the cards when trying to complete the exercises.

Definition of Symbols:

D = the set of all the cards in the deck of the game SET
R = the set of red cards
G = the set of green cards
P = the set of purple cards
1 = the set of cards with one shape
2 = the set of cards with two shapes
3 = the set of cards with three shapes
o = the set of cards with ovals
~ = the set of cards with squiggles
^ = the set of cards with diamonds
L = the set of cards with light shading
M = the set of cards with medium shading
H = the set of cards with heavy shading

Cardinality

A set, in general, is any collection of objects. One of the most basic ways we have of describing sets is cardinality. Cardinality is simply the number of elements or objects in a set. Another name for the cardinality of a set is the set's cardinal number. We use the symbol |X| to mean the cardinality of a set or cardinal number of some set X. For example, |R| means the number of objects (or cards) in the set R, which we have defined as the set of cards that are red in the game SET.

$|R| = 27$
$|M| = 27$
$|1| = 27$

Union

Union is a set operation, or a way of relating two sets together. The easiest way to think of union is that for any two sets, their union includes all of the elements that are in one or both of the sets. The symbol for union is $\cup$. When you think of union, you should think of the word "or". For example (R $\cup$ ~) means all the cards that are red or squiggles or both.

We can also use cardinality with union. For example, | R $\cup$ ~ | = 45, because there are 27 red cards and 27 cards with squiggles which adds up to 54, but since there are 9 cards with red squiggles that are counted twice, we subtract the number of red squiggles (9). In other words, we add the number of elements in each set and then subtract the number of elements that the two sets have in common.

The empty set: $\varnothing$= the empty set; a set with no elements.

Exercises:
For each exercise:
a. Write in words what the symbols mean, and
b. give the cardinal number

Example:
G $\cup$ ^
a. the set of cards that are green or are diamond
b. (27 green cards) + (27 diamonds) - (9 green diamonds) |G $\cup$ ^| = 45

1. R $\cup$ ~

2. M $\cup$ 1

3. 2 $\cup$ o

4. P $\cup$ H

5. o $\cup$ P

6. L $\cup$ ~

7. R $\cup$ 3

8. P $\cup$ G $\cup$ R

9. 1 $\cup$ 2 $\cup$ 3

10. ~ $\cup$ ^ $\cup$ o

Intersection

Intersection is another set operation or way of relating two sets together. The intersection of two sets is the elements that are in both sets, or the elements the two sets have in common. The symbols for intersection is ∩. When you think of intersection you should think of the word "and". For example, (G ∩ 1) means the set of cards that are green and have 1 shape.

We can use cardinality with intersection. Let's say that we wanted to know how many cards are green and have one shape (G ∩ 1). We could find each of the cards and count them or we could use what we know about the game SET®. We know that there are three different shapes and for each shape there are three different shadings. Whether we count the cards or try to "think out" the problem, we come up with 9 cards.

Exercises
For each exercise:
a. Write in words what the symbols mean, and
b. give the cardinal number

Example:
G ∩ 1
a. the set of cards that are green and have one shape on them
b. |G ∩ 1| = 9

11. R ∩ ~

12. ^ ∩ 2

13, G ∩ P

14. H ∩ ~

15. o ∩ R

16. P ∩ 3

17. R ∩ 1 ∩ o

18. G ∩ 2 ∩ ^

19. (R ∪ 1) ∩ ~ [hint: do what is in the parentheses first]

20. P ∩ (2 ∪ o)

Symmetric Difference

Symmetric difference is another set operation. The simplest way to think of symmetric difference is that it is all the elements that are in either one set or the other but not in both. The symbol for symmetric difference is Δ. Take the example (R Δ~). In words this means all the cards with red shapes or all the cards with squiggles, but not the cards with red squiggles. To find the cardinal number for (R Δ~) simply add the number of R (red cards) to the number of ~ (squiggles) then subtract the number of red squiggles. There are 27 red cards and 27 squiggles which adds up to 54. There are 9 red squiggles and since the red squiggles are in both the set of red cards and the set of squiggles, we must subtract them twice (54-18). Therefore the number of elements is 36.

Exercises

For each exercise:

a. Write in words what the symbols mean, and

b. give the cardinal number

***Example*:**

H Δ R

a. The set of cards that have heavy shading and cards that have one shape, but not the cards that have heavy shaded one shapes.

b. (27 heavy shaded cards) + (27 red cards) - (9 heavy shaded red cards from the set of reds) - (9 heavy shaded red cards from the set of heavy shaded cards) = 36 | H Δ R | = 36

21. P Δ G

22. ^ D o

23. 1 Δ 3

24. L Δ 1

25. ~ Δ 2

26. H Δ M

27. R Δ ~

28. (P $\cup$ R) Δ G [hint: remember to treat (P $\cup$ R) as one set]

29. (R $\cup$ G) Δ 2

30. (~ $\cap$ P) Δ ^

Complement

The complement of a particular set is simply all the elements in the universal set that are not in that set. When we are using the game SET®, the universal set is the whole deck of cards. Take the set P (purple cards). The complement of P (P') is all the cards that are not purple or, in other words, all the cards that are red or green. The cardinal number of P' (|P'|) is the number of elements in the universe (D) minus the number of elements in P.

|D| - |P| = |P'|
81 - 27 = 54

Exercises

For each exercise:
a. write in words what the symbols mean, and
b. give the cardinal number

***Example*:**
(R ∩ ~)'
a. all the cards that are not red squiggles
b. |D| - |(R ∩ ~)| = (R ∩ ~)'
81 - 9 = 72

31. ~'

32. 2'

33. H'

34. (H ∩ 1)'

35. (P ∩ ^)'

36. D'

37. (G ∪ o)'

38. (R ∪ L)'

39. (R ∪ G ∪ P)'

40. (R Δ 1)'

TIPS ON USING SET® IN THE CLASSROOM

By Patricia J. Fogle, Ph.D., D.O.

SET has been shown to be a valuable tool in the classroom for developing skills in logic, visual perception, and pattern recognition. Several tips gleaned from teachers and others should make **SET** easier to introduce to special groups.

First Introduction:

SET has four attributes (shape, shade, color and number) in the entire game (81 cards). List these attributes on the side board. Draw the shapes and shades, along with their appropriate names. List the numbers and colors for each attribute. Keep these in sight for students to refer to until they have a grasp of the characteristics they need to track.

Using actual cards, a transparency of a table of **SET** Cards, or the **SET** transparencies, introduces the concept that **each** attribute individually, must be all the same on all cards or must be all different on all cards. Usually six to ten examples will be sufficient to grasp the rule of **SET**.

By eliminating one of the attributes – e.g. color – one can more easily locate ***SETs*** in a smaller deck of cards (27 cards). Once students have mastered finding ***SETs*** in the smaller deck, all cards can be mixed again for the ultimate challenge.

Ages 4-6:

Smaller decks are used (as in First Introduction), but the attribute can be varied from one game to another. In one game kids might play with all solid cards, and in another game they might play with all squiggles, or all twos, or all greens.

The two hardest concepts to convey to this age group are the naming of the shapes used, and the meaning of shading (as opposed to color). Shapes may be defined in their usual manner, and be given an alternative name – e.g., the squiggle might be called a peanut or a worm. With regard to shading, one teacher said her kids referred to shading as "the guts", or what's on the inside – empty, partially filled, or full. Use the names that are appropriate and understandable to your kids.

Mental/behavioral handicaps:

The game is played as first introduced to the 4-6 year olds. In addition, if students lack the attention span to sort through each characteristic needed, two cards can be pulled out (for which the answer is on the remaining board) and placed side by side. One of the characteristics need on the third card is identified, then a second characteristic, then the third characteristic – and each time the previously identified characteristic is renamed. When all characteristics have been identified, the student is asked to locate the card which has all those characteristics.

Another approach can be used for those with more severe attention span/concentration difficulties. The game is started as above. As each needed characteristic is identified, the student is asked to identify each card which satisfies that characteristic, and to turn over any card which does not meet the needed characteristic. All characteristics are identified in similar fashion. Only one card will remain face upward – the solution to the puzzle!

Color blindness:

For colorblindness a deck of cards may be marked where all the red cards have a single black line down the side of the cards, the green cards have a single black line along the top and bottom of the cards and the purple cards are unmarked.

Deafness:

The easiest way to teach students with hearing difficulties how to play **SET** is have an individual versed in the rules of **SET** team up with a person gifted in using sign language. If possible, the two should discuss the characteristics of shape and shading prior to the instruction, so a clear message can be conveyed to the students. And the students can announce finding a ***SET*** by hand signal, if necessary.

Use of transparencies:

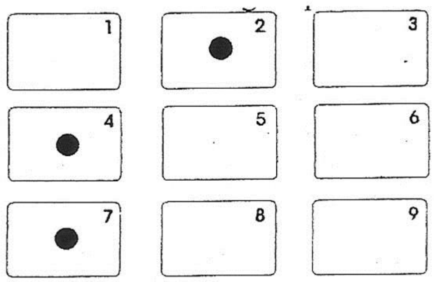
Transparencies are a great way to teach the game to large numbers at one time. They are also a way to put a **SET** daily Puzzle up so that students can do it first thing in the morning, which gets them into their seats, quiet, and thinking. The **SET** Puzzles are also great for individual stations when work is finished. It is helpful to label the position of each card with a number from one to twelve, and have the student identify the ***SET*** using the numbers, then descriptions. Another method of locating cards used in a ***SET*** is the grid system – rows might be 1, 2, 3, 4 and columns might be A, B, C. Cards would then be identified by letter-number combination, prior to the description of the card.

SET® AND MATRIX ALGEBRA

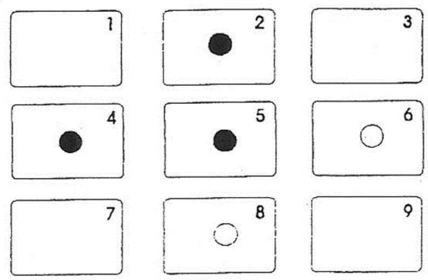
By Patricia J. Fogle, Ph.D., D.O.

The following two tables represent ways of aligning SET cards on a tic-tac-toe type board to make a magic square of *SET*s.

A



B

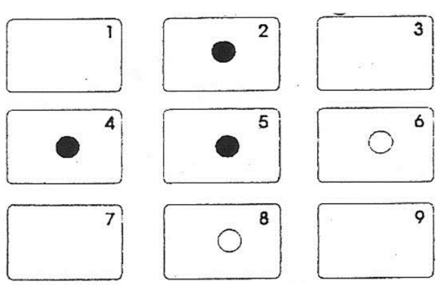


In both tables three SET cards are selected that in themselves do not make a *SET*. These cards are arranged on the board so that two of the cards are in a line, and the third card is laid anywhere except the third position in that line. When the two cards in that line are known, the third card in that line can be determined by the rules of SET. That card now forms a relation with the other card on the board, and the third card in that line can now be determined. Subsequently the entire board can be filled in.

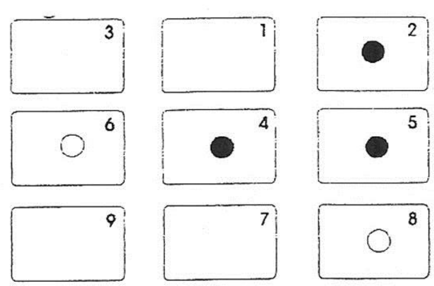
In Table A cards #4 and #7 define card #1. Then cards #1 and #2 define card #3. Cards #3 and #7 define card #5, and so on.

In table B cards #2 and #5 define #8. Cards #4 and #5 define #6. Now it does not appear that two cards are in a line. But this table also has a unique solution. Two methods are available to finish the puzzle: trial and error (until every row, column, and diagonal makes a *SET*), or using a concept from matrix algebra.

B



B'



In the above two tables matrix B has been transformed into matrix B'. In matrix algebra the absolute value of matrix B is negative the absolute value of matrix B' (/B/ = - /B'/) – any row or column transformation will work. In this transformed state one can now see that cards #2 and #4 will define card #9, and so on. The transformation can now be returned to the original table.

By doing a series of row and column transformations one can then see that four additional *SETs* that are not so obvious on the original board. In addition to each row, column, and diagonal being a *SET*, the four additional patterns are: 1) cards #2, #4, and #9; 2) cards #2, #6, and #7; #) cards #1, #6, and #8; 4) cards #3, #4, and #8 – a total of twelve *SETs* in nine cards.

SET® AND STATISTICS

By Patricia J. Fogle, Ph.D., D.O.

The use of statistics pervades the world in which we live. It is used arguably to defend positions in basic and applied scientific research, and ultimately affects all aspects of our lives. It therefore is important to understand the rationale and meaning of these "numbers" that affect our lives.

An easy way for students to begin to grasp the value of numbers involves collection of data while playing the game SET: The Family Game of Visual Perception. SET lends itself to this task because patterns which are removed from the board during play can be neatly categorized according to characteristics listed in Table 1 below.

Table 1: CATEGORIES OF *SETs*

ONE DIFFERENCE:

a) different:	shape	same:	shade, color, number
b)	shade		shape, color, number
c)	color		shape, shade, number
d)	number		shape, shade, color

TWO DIFFERENCES:

a) different:	shape, shade	same:	color, number
b)	shape, color		shade, number
c)	shape, number		shade, color
d)	shade, color		shape, number
e)	shade, number		shape, color
f)	color, number		shape, shade

THREE DIFFERENCES:

a) different:	shape, shade, color	same:	number
b)	shape, shade, number		color
c)	shape, color, number		shade
e)	shade, color, number		shape

FOUR DIFFERENCES:

a) different:	shape, shade, color, number

Problems which can be addressed by collecting data after the conclusion of a game include the following:

1)

(a) Sort each *SET* removed during play into its appropriate category by either number of differences or type of differences (according to the table above). Tally the totals in each category. Then calculate the percentages (or fractions) in each category.

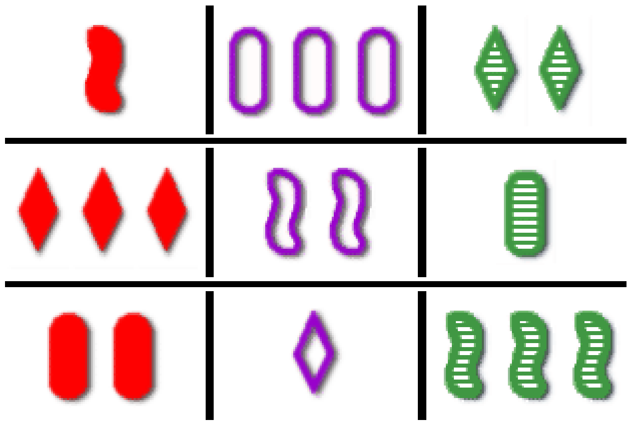
(b) After several games have been played add the totals in each category and calculate percentages again. Compare the values in this larger population to any individual game. The students may easily see the differences in "small population" vs. "large population" statistics.

2)

(a) Using two decks of identically arranged SET cards (after random shuffle of one deck) have one team play one deck of cards while another team plays the second deck. Stack each *SET* until the end of the game, and then sort each *SET* into the appropriate category. Compare responses from each team. Are there any differences? Since the boards started identically and cards were replaced in identical order, any differences would begin to demonstrate the effect of previous decision-making on subsequent possibilities.

(b) Randomly shuffle the deck. Lay out a board of the first 15 cards. Find all *SETs* within that board. Once the students believe they have located all the *SETs*, have them align all the cards using one attribute for sorting. For example, sort all the cards with three in one column, two in another, and one in another. Systematically check the cards to locate all *SETs*. Did the students miss any *SETs*, and if so, what category did they belong?

MAGIC SQUARES

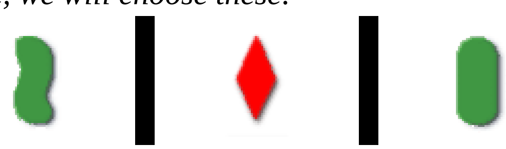


What you see here is a magic square, much like the addition and subtraction squares you may have used as a child.

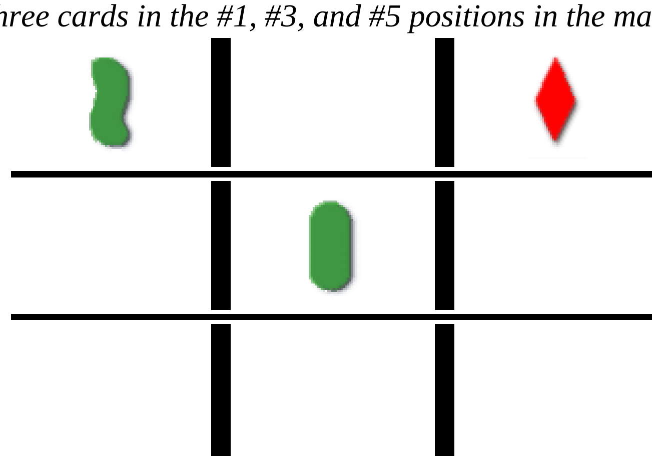
These magic squares are even more talented, as they all follow the rules of the card game SET®. To learn how to make one with ease, read on.

SET® cards contain four properties: **color**, **shape**, **number** of objects, and **shading**. The rules state for each property, they must all be equal, or all different. For example, if we look at the top row of the square, we see three different colors, three different shapes, three different numbers, and three different types of shading within the objects. Need more examples? Any line on the magic square yields a set. Constructing a magic square may seem complex at first glance, but in reality anyone can make one by following this simple process:

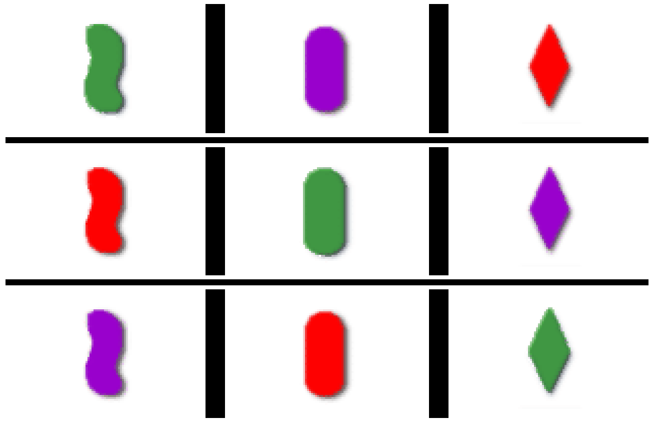
- *Choose any three cards that are not a set. (It will work with a set but the square becomes redundant) For example, we will choose these:*



- *Now place these three cards in the #1, #3, and #5 positions in the magic square.*



- *Using our powers of deduction, we can conclude that in order to create a set in the first row, the #2 card needs to have a different color, different shape, same number, and same shading as* *the #1 and #3 cards. That leaves us with a solid purple oval. The rest of the square can be completed in the same way, giving us the following magic square:*



A few examples will convince you that this method works. Not only does the magic square work but it can be theoretically proven through a mathematical model. This model makes an easy proof of the magic square as well as answers any questions about how SET® works.

MATHEMATICAL PROOF OF THE MAGIC SQUARES

By Llewellyn Falco

One day, while sitting by myself with a deck of SET® cards, I began to wonder whether or not I could construct a 3x3 square which made a set regardless of which direction you looked. I sorted the deck into single colors, and then started constructing a square. To my surprise it worked. I tried to make another one. It worked. As a test, I made a 3x3 square with all three colors, and *SETs* involving no similarities, and other *SETs* with only one difference. When it ended up working out I was convinced that no matter which cards you started with, you could always construct a 3x3 square that made a *SET* in every direction. Being an educated man, and a lover of mathematics, I decided that I should be able to prove this theory. So I set out to work; this is the fruit of my labor... First, we need a convention in which to label the cards. Thus, if we look at each characteristic on each card separately, and denote all variation to 1, 2, or 3...

Number[X_1] Color[X_2] Symbol[X_3] Shading[X_4] $\in\{1,2,3\}$

So the vector x=[p,q,r,s] completely describes the card.

For example: the card with one, red, empty, oval might be Number[X_1] = 1, Color[X_2] =1, Symbol[X_3] = 1, Shading[X_4] = 1, or x = [1,1,1,1].

For shorthand, I use the notation C_x to represent the card.
Where C_x = Number[X_1] , Color[X_2] , Symbol[X_3], Shading[X_4], and x = [p,q,r,s].

If I wanted to make the third card which makes a set from two cards Ca and Cb, I would have the card $C_{(ab)}$ where ab = [a_1b_1, a_2b_2, a_3b_3, a_4b_4]

and the rule for the operator is: If an = bn, then bn = xn and an = xn If an≠bn, then bn≠xn and an≠xn

For Example: 1*1=1, 1*2=3, 1*3=2
[1,1,2,2] [1,2,2,3] = [1,3,2,1]

This holds consistent with the rules of SET. If the first two cards are red the third must also be red; if the first one is a squiggle, and the second a diamond, the third must be an oval.

Here are some basic theorems in this group and their proofs:

<table>
<tr><td rowspan="3">$a_n * b_n = b_n * a_n$</td><td colspan="2">Proof 1.1
Two cases: $a_n = b_n$, $a_n \neq b_n$</td></tr>
<tr><td>Case 1:
$a_n = b_n$
1*1 = 1*1
1 = 1</td><td>Case 2:
$a_n \neq b_n$
1*2 = 2*1
3 = 3</td></tr>
<tr><td colspan="2">Note: this just shows that any two cards make a third, regardless of order.</td></tr>
</table>

<table>
<tr><td>$(a_n * b_n)\, c_n \neq a_n * (b_n * c_n)$</td><td colspan="2">Proof 1.2
$(a_n * b_n)\, c_n \neq a_n * (b_n * c_n)$
$3(2 * 1) = (3 * 2) * 1$
$3 * 3 = 1 * 1$
$3 \neq 1$</td></tr>
<tr><td rowspan="5">$(a_n * c_n)\,(a_n * b_n) = a_n(c_n * b_n)$</td><td colspan="2">Proof 1.3
There exist four options:
$a = b = c$
$a = b,\ b \neq c$
$a \neq b,\ b = c$
$a \neq b \neq c$</td></tr>
<tr><td>$a = b = c$</td><td>$(1*1)(1*1) = 1*(1*1)$
$(1*1) = (1*1)$
$1 = 1$</td></tr>
<tr><td>$a = b,\ b \neq c$</td><td>$(1*1)(1*2) = 1*(1*2)$
$(1*3) = (1*3)$
$2 = 2$</td></tr>
<tr><td>$a \neq b,\ b = c$</td><td>$(1*2)(1*2) = 1*(2*2)$
$3*3 = 1*2$
$3 = 3$</td></tr>
<tr><td>$a \neq b,\ b \neq c$</td><td>$(1*2)(1*3) = 1*(2*3)$
$3*2 = 1*1$
$1 = 1$</td></tr>
<tr><td rowspan="3">$a(a * b) = b$</td><td colspan="2">Proof 1.4
Two cases exist: $a = b$ or $a \neq b$</td></tr>
<tr><td>Case 1:
If $a = b$
then $1(1*1) = 1$
$1*1 = 1$
$1 = 1$</td><td>Case 2:
If $a \neq b$
Then $1(1*2) = 2$
$1*3 = 2$
$2 = 2$</td></tr>
<tr><td colspan="2">Note: This is just a case that a set of three cards can be made by taking any two of the three cards a*b=c means b=c*a means a=b*c, now we see that (a*b) =a*b, then move to the other side…a(a*b)=b</td></tr>
</table>

The Square

So let us begin by choosing any three cards: a, b, and c, and placing them in positions 7, 5, 9.

1	2	3
4	**C_c**	6
C_a	8	**C_b**

Now we need to fill in the blanks for the remaining cards. Starting with card 8; it needs to complete the *SET* with the cards **C**a and **C**b. We now look at the multiplier. The new card will be the product of **C**a operating on **C**b which is **C**ab. Likewise, filling in slots 1 and 3 leaves us with the square below.

C_{bc}	2	**C_{ac}**
4	C_c	6
C_a	**C_{ab}**	C_b

We now know that we have three *SETs* on the board (7,8,9; 7,5,3; 1,5,9). However, how should we go about filling slot two? I chose to combine 5 and 8 and give slot 2 the card **C**$_{c(ab)}$. Now I have a *SET* going down 2,5,8, but the theory states that 1,2,3 should form a *SET* as well. This means if I choose to combine **C**$_{bc}$ with **C**$_{ac}$, it would equal **C**$_{(bc)(ac)}$, which must be the same card as **C**$_{c(ab)}$. Therefore we must show that:

(bc)(c(bc)) = (ac)	
c(b(ab)) = ac	by 1.3 and 1.1
c(a) = ac	by 1.4 and 1.1
ac = ac	by 1.1

Now we fill in the two remaining slots of 4 and 6 by combining down to end up with 4 equaling **C**$_{a(bc)}$ and 6 equaling **C**$_{b(ac)}$. So now we have the following square:

C_{bc}	**$C_{c(ab)}$**	C_{ac}
$C_{a(bc)}$	C_c	**$C_{b(ac)}$**
C_a	C_{ab}	C_b

Now that all other rows, columns, and diagonals have been accounted for, we only have to prove that 4,5,6 is a *SET*. This means

(a(bc))*c = b(ac)

b(bc) = c	by 1.4
(a(bc))(b(bc)) = b(ac)	substitution of b(bc) for c
(bc)(ab) = b(ac)	by 1.3
(bc)(ba) = b(ac)	by 1.1
b(ca) = b(ac)	by 1.3
b(ac) = b(ac)	by 1.1

This completes the proof of the square. Four *SETs* still remain unaccounted for (1,6,8; 3,4,8; 7,2,6; 9,2,4). We note that if we prove one of these all must be true since now we can reconstruct this square by placing any card from 1,3,7, or 9 in the beginning slot and still get the same square.

Proof of the *SET* 1,6,8

C_{bc}	**$C_{c(ab)}$**	C_{ac}	(bc)(ab) = b(ac)(bc)
$C_{a(bc)}$	C_c	**$C_{b(ac)}$**	(ba) = b(ac)
C_a	C_{ab}	C_b	b(ac) = b(ac)

Proof of the *SET* 3,4,8:

C_{bc}	$C_{c(ab)}$	**C_{ac}**	(ac(a(bc)) = ab
$C_{a(bc)}$	C_{c}	$C_{b(ac)}$	a(c(bc)) = ab
C_{a}	**C_{ab}**	C_{b}	a(b) = ab

Proof of the *SET* 7,2,6:

C_{bc}	**$C_{c(ab)}$**	C_{ac}	a(c(ab)) = b(ac)
$C_{a(bc)}$	C_{c}	**$C_{b(ac)}$**	(ab))(c(ab)) = b(ac)
C_{a}	C_{ab}	C_{b}	(ab)(bc) = b(ac)
			b(ac) = b(ac)

Proof of the *SET* 9,2,4:

C_{bc}	**$C_{c(ab)}$**	C_{ac}	(b(c(ab)) = a(bc)
$C_{a(bc)}$	C_{c}	$C_{b(ac)}$	(a)(ab))(c(ab) = a(bc)
C_{a}	C_{ab}	**C_{b}**	(ac)(ab) = a(bc)
			a(bc) = a(bc)

The largest group of cards you can put together without creating a *SET* is 20. By following this method, you'll understand how.

If we choose just two characteristics of a card (for example, shape and number), we can then plot it on a matrix.

	1	2	3
Squiggle	●		
Diamond			
Oval			

So now we can see if three cards form a set by noticing if they make a line on the matrix, as shown below. These three cards all have one object and different shapes.

	1	2	3
Squiggle	●		
Diamond	●		
Oval	●		

Similarly, the following lines all produce sets, even if they wrap around the matrix in space like the one on the far right.

	1	2	3
Squiggle			●
Diamond		●	
Oval	●		

	1	2	3
Squiggle			
Diamond			
Oval	●	●	●

	1	2	3
Squiggle		●	
Diamond			●
Oval	●		

Now the following matrix shows us how many squares we can fill without creating a line: **4**.

	1	2	3
Squiggle	●		●
Diamond			
Oval	●		●

Now we can add a third characteristic, color...and we can think of the matrix as a depiction of a 3D tic tac toe board. You can see below how to plot 9 cards without a SET.

	1	2	3
Squiggle	●		●
Diamond			
Oval	●		●

Red

	1	2	3
Squiggle		●	
Diamond	●		●
Oval		●	

Green

	1	2	3
Squiggle			
Diamond		●	
Oval			

Purple

[continued on next page]

When using all four SET card properties, we can plot a 20 card NO SET.

	1	2	3
Squiggle	●		●
Diamond			
Oval	●		●

Red

	1	2	3
Squiggle		●	
Diamond	●		●
Oval		●	

Green

	1	2	3
Squiggle			
Diamond		●	
Oval			

Purple

solid

	1	2	3
Squiggle		●	
Diamond	●		●
Oval		●	

Red

	1	2	3
Squiggle	●		●
Diamond			
Oval	●		●

Green

	1	2	3
Squiggle			
Diamond		●	
Oval			

Purple

striped

	1	2	3
Squiggle			
Diamond		●	
Oval			

Red

	1	2	3
Squiggle			
Diamond		●	
Oval			

Green

	1	2	3
Squiggle			
Diamond			
Oval			

Purple

open

THREE WAYS TO USE SET® IN THE CLASSROOM

Classroom Warm Up Using SET®

Note: The following exercise can be done with the use of SET transparencies or an interactive whiteboard.

Get Students Seated and Ready to Work

Go to www.setgame.com and view the SET Daily Puzzle. Duplicate the puzzle on the overhead or project it onto the interactive whiteboard. Have the students find the six *SETs*. This can be done as a whole class activity with students raising their hands and the teacher recording the *SETs* on the board. Or it can be a quiet activity with the students writing the *SETs* on a piece of paper. Alternatively, the puzzle could be printed out for each student to complete at their desk.

Additional daily puzzles can be found on the NYTimes website: www.nytimes.com/set.

Variation: Have students make up a puzzle that contains 6 *SETs*.

Rules for Playing SET® with Teams in the Classroom

Note: The following exercises call for the use of SET transparencies or an interactive whiteboard.

Divide the class into teams of 3-8 students each. Go to www.setgame.com and view the SET Daily Puzzle. Duplicate the puzzle on the overhead or project it onto the interactive whiteboard.

Quiet Team Play

The members of each team must work together to determine all 6 *SETs* in the Daily Puzzle. One person on the team writes down each *SET* as it is found. The team leader raises his/her hand when the team has found all six *SETs*. The first team to correctly identify all 6 *SETs* wins.

Quiet Team Play to Develop English Language Skills

When the focus of the exercise is for developing English skills, the students work together to write full sentences describing each *SET*, rather than allowing them to use numbers to describe the cards. For example, "The first *SET* consists of one open red oval, two solid red ovals, and three shaded red ovals. The second SET consists of one open red oval, one solid red oval, … etc." The team leader raises his/her hand when the team has found all six *SETs*. The first team to correctly identify all 6 *SETs* wins.

Team Play to Develop Communication Skills

Each team has a monitor who stands. The team members sit in front of the monitor. When a *SET* is seen, the team member raises his/her hand, and then the monitor raises his/her hand. The teacher calls on the monitor, who asks the team member what the *SET* is, then the monitor tells the teacher. If any team member speaks out of turn, he/she must sit out for the next turn. This encourages the quiet ones to speak up, keeps the noisy ones quiet and develops communication skills.

Language Skills Using SET® in the Classroom

For this exercise, transparencies can be placed on the overhead projector or printed on a worksheet for students to work individually or in teams.

Activity:
Place two cards on the overhead projector. Ask the students to describe the missing card.
For younger students, have them fill in missing adjectives in a sentence you provide.

More Advanced Activity:
Place two cards on the overhead projector. Ask the students to draw the missing card and then write a sentence describing it. *Example:* I need two open purple ovals to complete this *SET*.

Place two new cards on the overhead projector. Ask the students to draw the missing card and then write a sentence using a different verb or sentence structure. *Example:* In order to complete this *SET*, a solid red oval is required.

Continue as above: The third sentence could be: “Please give me an open red diamond.”

DEVELOPING MATHEMATICAL REASONING USING ATTRIBUTE GAMES

Reproduced with permission from the ***Mathematics Teacher***, copyright 1999 by the NCTM

By Anne Larson Quinn, Ph.D., Associate Professor, Edinboro University, Quinna@edinboro.edu
Frederick Weening, Ph.D., Assistant Professor, Edinboro University, Fweening@edinboro.edu
Robert M. Koca, Jr., Ph.D.

The game of SET® has proven to be a very popular game at our college mathematics club meetings. Since we've started playing, the membership has grown every month. In fact, one of our members brought her six year old son to a meeting, and he now looks forward to playing SET® with us every month. As a result of playing the game in our club and thinking about the results, we created and solved a variety of mathematical questions. For example, we wondered about possible strategies for winning and conjectured about phenomena that happened when playing. These questions involve a wide variety of traditional mathematical topics, such as the multiplication principle, combinations and permutations, divisibility, modular arithmetic, and mathematical proof.

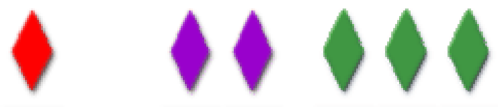
In addition to encouraging the posing and solving of these problems in our math club, we took the game and our questions into our classrooms to see what reasoning could be encouraged. We tested our original questions on several groups of junior high and high school students and on several hundred freshmen and sophomore college students who were not mathematics majors.

The purpose of this article is to show how games such as SET® can be used to develop mathematical reasoning by describing student investigative work that has resulted from playing the game. After giving a description of the game, we will pose and answer some of the questions that were solved by members of our club and by students ranging in academic level from ninth grade to college. We will also describe what teachers can do to facilitate the development of reasoning using this game. Although this article discusses problems that were generated from the game of SET®, any game that uses attributes can be used to stimulate logical thinking.

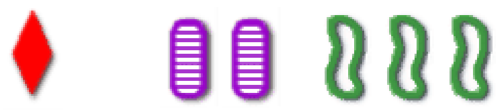
Introduction to the game of SET®: Twelve cards are placed face up on the table. Each player looks for a "SET" of three cards. When players find *SETs*, they put them in their own piles, and three new cards are put on the table to bring the total back to twelve. The game continues until all of the cards are dealt and no more *SETs* are found. If at any time there are no *SETs*, three more cards are added until a *SET* can be found. The player with the most *SETs* at the end of the game wins.

The deck of cards: Each card can be identified by four attributes, each of which has three values: number (1, 2, 3), color (red, green, purple), symbol (diamond, oval, squiggle), and shading (open, striped, solid). The deck is made up of one card of each type. For example, there is one card that has 1 red diamond with solid shading. (We will call it: one-red-diamond-solid.)

Sets: Three cards make a set if, for each attribute, the values on the cards are either all the same or all different. To illustrate, consider the following example.

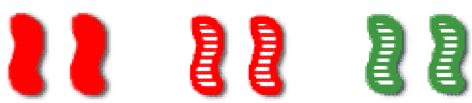


a



b

c



d

Figure 1: Examples and non-examples of *SETs*

The three cards in Figure 1a are a *SET* because their symbols are all the same (diamonds), their shadings are all the same (solid), their numbers are all different (1, 2, 3), and their colors are all different (red, purple, green). The three cards in Figure 1b are a *SET* because their numbers are all different, their colors are all different, their symbols are all different, and their shadings are all different. However, the three cards in Figure 1c are not a *SET* because their symbols are neither all the same nor all different (2 diamonds and 1 oval). Similarly, the three cards in Figure 1d are not a *SET* because the colors are neither all the same nor all different (2 red and 1 green). Thus, for any attribute, if the values are the same for exactly 2 of the 3 cards, the 3 cards are not a *SET*.

A summary of the questions we posed is seen in Table 1.

	QUESTION
#1	Find as many *SETs* as possible in Figure 2.
#2	How many cards must be in the deck?
#3	How many *SETs* (including overlapping ones) are possible in the deck?
#4	What is the best strategy when searching for *SETs*? Which type are you most likely to find?
#5	What is the average number of *SETs* among 12 randomly selected cards?
#6	If one attribute is fixed, how many cards could there be that contain no *SETs*?
#7	Find as many cards as possible that contain no *SETs*.
#8	Can only three cards be left at the end of the game?

Table 1:
Questions posed to our junior high, high school, and beginning college students

Question #1: Find as many *SETs* as possible in Figure 2.

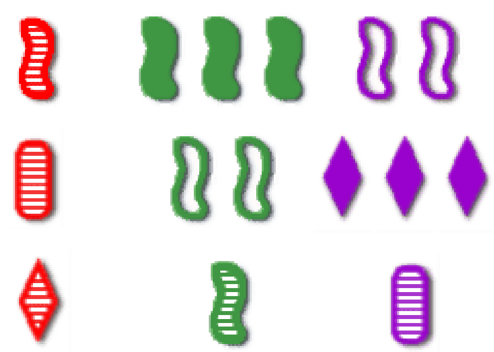


Figure 2: Find as many *SETs* as you can

Answer #1: There are six *SETs*, one in every row and one in every column.

Student Work on #1: While searching for *SETs*, students developed several strategies that also helped them answer the later questions. Sara, a ninth grade algebra student, started by looking at two cards and then determining what the third card must be to form a *SET*. Corey looked only for *SETs* that had cards that differed in number. Kathleen scanned the entire group of cards for similarities. For example, she looked for *SETs* among the solid cards.

While playing once, they saw that there were no *SETs* in the last 12 cards, as seen in Figure 3. The students built on the above strategies in order to try to prove this. Corey said that there was only one card with 1 figure left and that it did not form a set with any of the other cards. Kathleen said that there was only one striped card. Since it didn't form a *SET* with any of the other cards, then any possible *SET* must be made of three solid cards or three open cards. Similarly, since the one squiggle card didn't form a *SET* with any of the other cards, possible *SETs* must be all diamonds or all ovals.

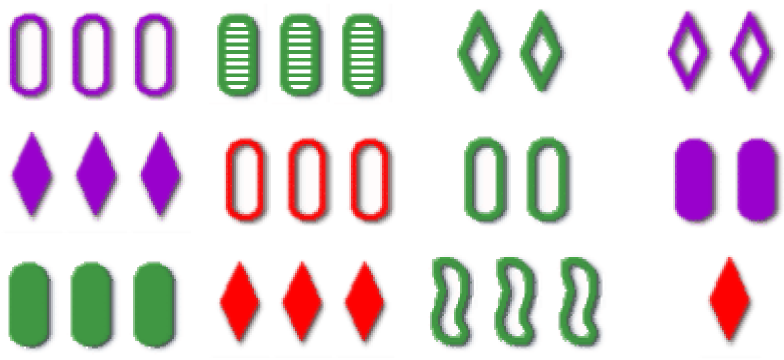


Figure 3: Prove that there are no *SETs* among these cards

Some students had difficulty playing or answering questions if they considered all four attributes. Initially, this frustration can be reduced by considering only the red cards.

Question #2: How many cards must be in a deck?

Answer #2: Since each of the four attributes has three values, there must be 3^4=81 cards.
Student Work on #2: Students at all levels had no problem with this question. Most of the ninth grade students solved this problem like Sara did. She started with one card: one-red-oval-open. She then considered the cards that differed from this one in shape and then in number. She drew these 9 cards, as in Figure 4. Then she said these could also be striped or solid, hence 9x3=27, and that the 27 cards could also be green or purple, hence 27x3=81.

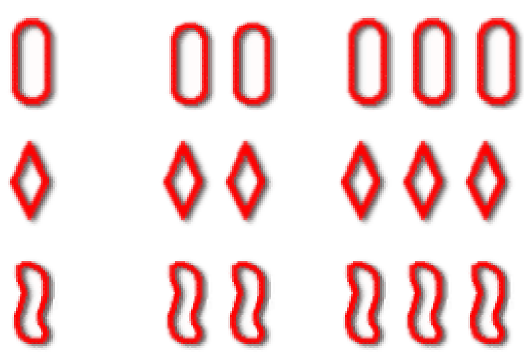
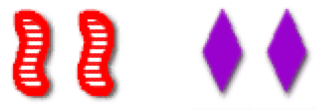


Figure 4: There are 9 open-red cards

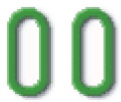
Most of the college freshmen started solving the problem by constructing a tree diagram, where attributes were considered one at a time. The result was 3x3x3x3=81 cards.

Question #3: How many *SETs* (including overlapping ones) are possible in the deck?

Answer #3: For each of the 81 cards, any of the remaining 80 could be used to make a *SET*. Once these two are chosen, only one card exists to complete the *SET*. In fact, a good exercise to improve one's speed is to take two cards and name the third card which completes the *SET*. To illustrate, if the two cards are the ones seen in Figure 5a, the numbers are the same (two), the colors are different (red, purple), the symbols are different (squiggle, diamond), and the shadings are different (striped, solid). So, the third card is the one in Figure 5b (two-green-oval-open).



a



b

Figure 5: Two cards in a *SET* determine the third

So there are 81x80x1 = 6480 possibilities. However, since the order of the cards in the *SET* doesn't matter, there are 6480/3! = 1080 possible sets.

Student Work on #3: Sara knew that once the first two cards had been selected that the third card of the *SET* was determined. She started this problem by focusing on one card: one-open-green-diamond. She noted that the second card could be one of the following cards: two-open-green-diamond (varying the number), one-open-red-diamond (varying the color), one-open-red-oval (varying the symbol), or one-striped-red-diamond (varying the shading). She concluded in a matter of minutes that the first card could be chosen 81 ways, the second card 4 ways, and the third card one way. So she answered 81x4 = 324. Corey and Kathleen noticed that the second card could be chosen in more than 4 ways, by changing more than one attribute at a time. They spent several minutes coming up with as many second cards as they could. After becoming frustrated with drawing out all of their second cards, Kathleen realized it would be any of the remaining 80 cards. Corey concluded there would be 81x80=6480 *SETs*. The teacher asked if they counted any of the *SETs* more than once. They looked at one sample *SET* and saw that by moving the cards around that it would have been counted 6 times. So Corey concluded 81x80/6 = 1080.

Some of the college students took considerably more time than the ninth graders to correctly count the total number of *SETs*. Although early on Kathryn and Angela, college sophomores, realized that the second card could be any of 80, they became very concerned about counting the same set more than once. In their attempt to avoid overcounting, they systematically arranged all of the 81 cards into 3 groups, according to symbol. By doing this, they were actually building ideas that would help them easily answer later questions, such as question #4. After correctly counting all of the *SETs* that could contain one fixed card, one by one, they got frustrated about whether they would overcount while generalizing to a problem that contained all 3 symbols. After the teacher suggested that they worry about overcounting at the end of the problem, they easily came up with the correct answer. Kathryn even made a connection between the 6 ways one *SET* could be written and the way she represented electron configurations in her chemistry class.

This was also a very good question to ask the college freshmen who had studied probability and statistics in class. Many initially suggested that the answer might be 81C3 (81!/[(81-3)!3!]= 85,320), the number of possible groups of three cards. When asked whether these were all *SETs*, they realized that each group of 3 did not necessarily make a *SET*. Some students then said the answer must be 81P3 (or, equivalently 81x80x79 = 511,920), since these were the two techniques they knew best. After discussion, they realized that this was not a straightforward problem. Playing the game was instrumental in solving the problem. Those that searched for SETs by starting with one card at a time (instead of scanning for general patterns) were most successful in arriving at: 81x80x1. Some needed to be asked whether they had overcounted before they thought to divide by 6. Working in groups, most of the class had success on this problem, and many commented that the problem really made them think.

Question #4: What is the best strategy when searching for *SETs*? Which type are you most likely to find?
Answer #4: On each of the four attributes, the values must be all the same or all different. So, *SETs* are of the following types: 4 different, 3 different and 1 same, 2 different and 2 same, or 1 different and 3 same. We'll determine how many of each type exist with the full deck of 81 cards.
For the "4 different" type, consider that the first card can be any of the 81. The second card must be different on all attributes. So there are 24 =16 possible cards (2 colors, 2 shadings, 2 symbols, 2 numbers). The third card is determined. As with the total number of cards, each *SET* has been counted 3!=6 times. So there are (81x16x1)/6=216 *SETs* of this type.

For the "3 different and 1 same" type, the first card can by any of the 81. The second card will stay the same on one of 4 attributes. If, for example, the common attribute is color, the second card can by any of 23 =8 (2 shadings, 2 symbols, 2 numbers). So the answer is (81x4x8)/6=432. For the "2 different and 2 same" type, again the first card can by any of 81. There are 4C2=6 ways to select the two attributes that remain the same for the second card. If, for example, these are color and symbol, there are 22 =4 ways to select the second card (2 shadings, 2 numbers). So the answer is (81x6x4)/6=324. The last type could be determined in a similar manner. The likelihoods change during the course of the game as *SETs* are removed. However, the likelihoods that hold at the beginning of the game are summarized in Table 2.

TYPE OF SET	WAYS TO PICK 1st CARD	WAYS TO PICK THE ATTRIBUTE THAT IS SAME ON THE 2nd CARD	WAYS TO PICK THE 2nd CARD	WAYS TO PICK THE 3rd CARD	NUMBER OF SETS OF THIS TYPE	LIKELIHOOD OF THIS TYPE OF SET
4 different/ 0 same	81	$_4C_0 = 1$	2^4 =16	1	(81x16)/6 =216	216/1080 =20%
3 different/ 1 same	81	$_4C_1 = 4$	$2^3 = 8$	1	(81x4x8)/6=432	432/1080 =40%
2 different/ 2 same	81	$_4C_2 = 6$	$2^2 = 4$	1	(81x6x4)/6=324	324/1080 =30%
1 different/ 3 same	81	$_4C_3 = 4$	$2^1 = 2$	1	(81x4x2)/6=108	108/1080 =10%
TOTAL					1080	

Table 2: Probabilities of different "types" of *SETs*

Student Work on #4: Students were very interested in posing and solving this problem since each of the students had commented while playing the game that certain "types" of *SETs* seemed more common. Working in a group, members of the math club solved this problem as shown above in about 35 minutes. They tried a few cases with the cards before they were able to come up with these general answers. They saved the hardest case (2 different and 2 same) for last and had already deduced the answer should be 1080-(216+432+108)=324. Jaclyn and Paul were very excited when they were able to prove why this answer was 324, as seen in Table 2. While solving the problem, Mike and Cathy posed questions such as: "Should the answer for 3 different be the same as 3 same?", and "Should 4 different be the same as 4 same?" (until they realized that "4 same" could not occur since there was only one of each type of card).

One group of ninth grade students (Sara, Kathleen, and Corey) came up with an answer similar to the one above in about 1 hour. The teacher suggested starting with the easier cases. For "4 different", they started with one card: one-open-red-diamond. Then they laid the cards out on the table and made every possible *SET* of this type with this first card, realizing that they couldn't chose a second card which had the same value on any of the 4 attributes. So the second card had to be green or purple, squiggle or oval, solid or striped. Instead of dividing by 6 at the end, the ninth graders didn't overcount. They let the second card have 2 figures, and then the third card was determined. So they came up with the 2 3=8 second cards systematically and concluded that the first card (having 1 figure) could be any of 27. So the answer was 27x8=216. They got each of the above answers, sometimes needing to be reminded that the different attributes could be any of 4, instead of just the one they were discussing. They left "2 different and 2 same" for last, and simply answered that it was 1080-(216+432+108)=324, as the math club had also conjectured. Then they proceeded to come up with the correct probabilities for each type. Note that none of these students had ever studied combinatorics and that there were only minor suggestions made by the teacher. The student's success in creating these general arguments seemed to come from the fact that they could use the cards to first create special cases and from the insight they had gained from noticing and looking for different types of *SETs* while playing the game.

Question #5: What is the average number of *SETs* among 12 randomly selected cards?

Answer #5: Given any two cards, there are 79 cards remaining and exactly one of these completes the *SET*. It follows that the probability of any three cards making a *SET* is 1/79. Since the number of possible combinations of three cards chosen from twelve cards is 12C3 (12C3 = 12!/[(12-3)!3!] = 220), the expected number of *SETs* in a group of 12 cards is 1/79 times 12C3, or approximately 2.78. However, note that since the expected number of sets includes overlapping sets that share cards, they cannot all be used when playing the game. In addition, the above argument assumes a full deck of 81 cards. Part way through a game, the results would change.

Student Work on #5: As a result of experience from playing, all groups guessed that the answer would be between 2 and 3. This correct conjecture that was gained from experience helped guide them towards a correct theoretical answer. In the math club, Cathy noticed that the probability of any 3 cards making a set was 1/79. Since 12 was 4 groups of 3, she concluded that the answer would be 4/79. They worked together on this (noting the answer should be between 2 and 3), and then Mike realized that there would be 12C3 possible groups of 3 cards (including overlapping ones). So he concluded 12C3 times 1/79, or approximately 2.78.

Vicki, a student in a freshmen statistics course who was familiar with the standard expected value formula [E(X)= S X * P(X)], had trouble figuring out that X would be 1 each time. The teacher suggested this and then asked how many different groups of 3 existed in the group of 12. This helped Vicki finish the problem correctly. Jaclyn and Rich, from the math club, also had to discuss what values X and P(X) would have. For those who knew this formula, the question was a good application of the formula. However, some in the math club, such as Mike, came up with the answer without knowing this formula.

Sara, the ninth grade student who had never studied probability, had realized while playing that the first two cards determined the third card. When considering any 3 cards, she quickly said that the chance the third card would complete the set was 1 in 79, but didn't complete the problem.

Question #6: If one attribute is fixed, how many cards could there be that contain no sets?

Answer #6: We proved that 10 cards that share one attribute value must contain a *SET*. However, this is somewhat beyond the scope of this article. So students were just challenged to find as many red cards (fixing the color attribute) that they could that contained no *SETs*. The upcoming student work shows that it is possible to find 9 cards.

Student Work on #6: The ninth graders started with a few red cards that contained no *SETs*. Then they took each of the other red cards from the deck, one at a time, and either tossed it out (if it produced a *SET*) or included it (if it didn't). They came up with 9, but realized they had made one mistake, so they had 8. They wondered if they could exchange the ninth card for one of the cards they had discarded. After several attempts with 9 cards, Corey noted that whenever they had 2 striped, 3 open, and 4 solid cards that they always found a *SET*. He made this conjecture after trying many different cases, and then he provided a partial proof as he demonstrated more cases. He then generalized on this and said that having 2 oval, 3 squiggle, and 4 diamond cards would be the same mathematically and would cause the same problem. (Corey's realization was actually a crucial conjecture that we used when proving the complicated theorem that 10 cards that had one common attribute value must contain a *SET*. Although a full discussion of the theorem is not included, it is interesting to note that thinking about this game inspired advanced reasoning in junior high and high school students.)

As a group, the ninth graders then tried to get 9 cards by arranging their cards according to attribute. For example, they considered arrays of 9 cards with: 3 of each of the 3 symbols, 3 of each of the 3 shadings, and 3 of each of the 3 numbers. However, these 9 cards always contained a *SET*. After they became frustrated, the teacher suggested that to avoid having 2 striped, 3 open, and 4 solid, they could consider 1 striped, 4 open, and 4 solid. After a few attempts, they came up with the following 9 cards that contained no *SETs*, as seen in Figure 6. Each attribute value occurred either 3, 3, and 3 times or 1, 4, and 4 times.

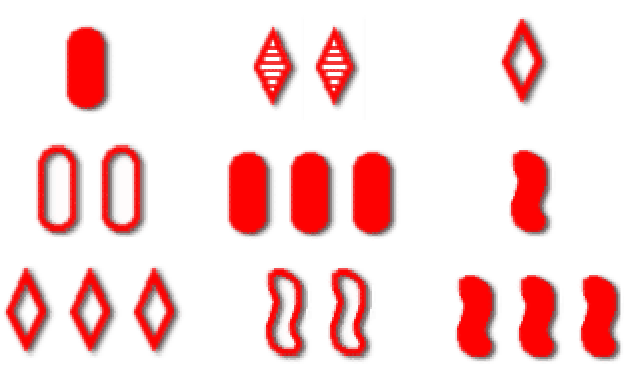


Figure 6 : Nine red cards which contain no *SETs*

Question #7: Find as many cards as possible that contain no *SETs*.

Answer #7: Miller (1997) and Set Enterprises, Inc. (1998) both illustrate that one can find 20 cards that contain no *SETs*. Miller uses 6 red, 8 green, and 6 purple cards. Both mention that they are currently working on a proof to show that 20 is a global maximum, but neither reports one.2 Both also discuss creating a computer program to answer this problem.

Student Work on #7: Students realized from experience that sometimes even 18 cards contained no *SETs*. However, since constructing the group of cards oneself can prove to be a challenge, we just asked students to find as large a number as they could.

Since the ninth graders (Sara, Corey, and Kathleen) had spent a lot of time with just the red cards, Sara quickly generalized on her answer for #6 and answered 9x3=27, since green and purple cards could also be included. However, Kathleen then noticed that they would then have three cards (1 red, 1 green, and 1 purple) that were one-open-oval, thus forming a *SET*. So Sara quickly responded that the 18 red and green cards would contain no *SETs*, as in Figure 7.

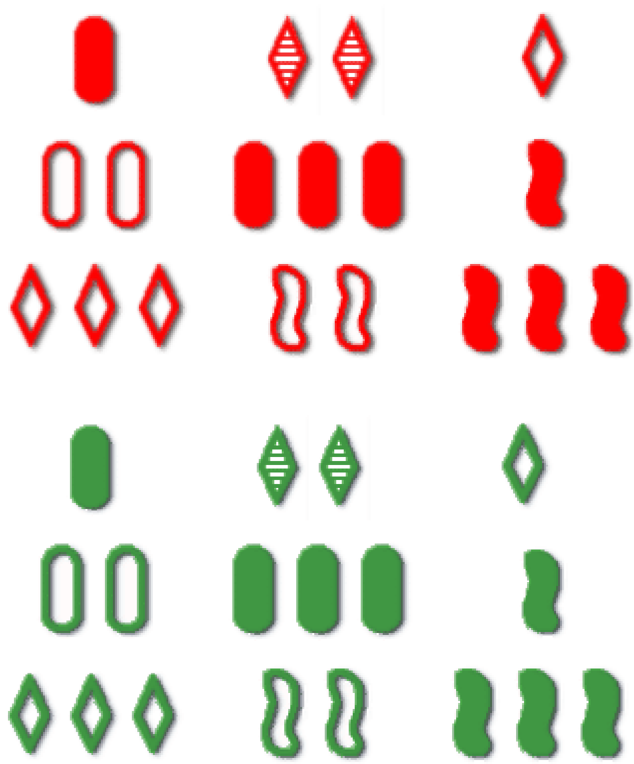


Figure 7: Eighteen cards which contain no *SETs*.

Kathryn and Angela, college sophomores, began this problem with cards of all three colors. They dealt 12 cards and then subtracted any of the cards that would force a *SET* to be found. Then they proved that the 9 cards they had left contained no sets by placing the existing cards into columns according to shading: solid, striped, and open. They noted that any *SET* would have to be contained in a column or would have one card from each column. They quickly went through the rest of the deck, including each card if it did not make a *SET* with any of the cards already on the table. They came up with 18 cards that contained no *SETs*, as seen in Figure 8. Their answer of 18 was a local maximum (since none of the discarded cards could be included), and we discussed how this concept related to calculus. Then Kathryn asked whether there would be another 18 cards that would contain no *SETs*.

Immediately she answered her own question by observing that she could change all the solid cards to striped, the striped to open, and the open to solid. Then she and Angela went on to count other similar ways these 18 could be altered. Kathryn was so inspired by thinking about the mathematics involved in SET® that she searched the Internet for unsolved SET® problems. She is currently excited about trying to prove one of these unsolved problems and is also thinking about pursuing a mathematics minor.

Figure 8: Eighteen cards which contain no *SETs*

Question #8: Can only three cards be left at the end of the game?
Answer #8: Although the game frequently ends with six or nine cards, we noticed that we never ended with three cards on the table and were inspired to investigate whether this would be possible. The answer proved to be no. To see this, consider the attribute of number. In total, there are 162 figures on the cards. (There are 27 cards with 1 figure, 27 cards with 2 figures, and 27 cards with 3 figures.) Three cards belong to a set with respect to number if and only if the sum of their figures is a multiple of 3. This is because the only possible *SETs* are three cards with 1 figure, three cards with 2 figures, three cards with 3 figures, or one with 1 figure and one with 2 figures and one with 3 figures. Therefore, if the previous 26 *SETs* are valid, the number of figures left for the last three cards will be 162-3 $\underline{k}$, which is a multiple of 3. Hence, the last three cards do form a *SET* with respect to number. Since the other attributes could be considered in a similar manner, the game cannot end in just 3 cards.

For another approach, consider the attribute of shading. At the beginning of the game, there are 27 solid, 27 striped, and 27 open cards. Notice that 27 º 0 mod 3. (27 is equivalent to 0 modulo 3. This

means 27 and 0 have the same remainder when divided by 3.) When a *SET* is taken from the deck, we will be left with either 24 of one shading and 27 of the other two shadings (27 º 24 mod 3) or with 26 of each shading (26 º 26 mod 3). After each *SET* is taken, the numbers of each shading continue to be equivalent modulo 3. So, to end with 2 of one shading and 1 of another would be impossible, because 2 ≠ 1 mod 3. So the last 3 will be a *SET* with respect to shading. Analyzing the other attributes similarly, the last 3 cards will be a *SET*.

Student Work on #8: To simplify the problem, the ninth graders played the game with only red cards. After seeing that the last 3 cards made a *SET*, they noted that among the last three cards, there were 3 cards with 3 figures, 3 cards with diamonds, and 1 card of each type of shading. They said that for the last three not to be a *SET*, there might be, for example, 2 solid and 1 striped. Corey noted that the cards started with 9 of each type of shading. They speculated about possible games that could happen with red cards and then with all cards. The number of each shading as each *SET* is taken are seen in Table 3.

SOLID	STRIPED	OPEN	SOLID	STRIPED	OPEN
9	9	9	27	27	27
6	9	9	26	26	26
6	6	9	23	26	26
3	6	9	23	23	26
3	3	9	23	23	23
3	3	3	20	23	23
2	2	2	20	20	23
1	1	1	20	20	20
0	0	0	17	20	20
			16	16	16
SOLID	STRIPED	OPEN	15	15	15
9	9	9	12	15	15
8	8	8	12	12	15
8	8	5	12	12	12
5	5	5	9	12	12
4	4	4	9	9	12
3	3	3	9	9	9
0	3	3	8	8	8
0	0	3	5	8	8
0	0	0	5	5	8
			5	5	5
			2	5	5
			2	2	5
			2	2	2
			1	1	1
			0	0	0

Table 3:
The ninth graders show why there cannot be 3 cards left at the end of the game

They discussed the patterns they saw, noting that either 3 was subtracted from one category or 1 from each category. Corey noted that the 27s at the beginning of the game were each divisible by 3. After some discussion, they realized that while the other numbers in the table were not always divisible by 3 that the numbers in each row always had the same remainder when divided by 3. Additional questions motivated by the game, some created by our students, can be seen in Table 4.

ADDITIONAL QUESTIONS
1. Prove that 5 cards that have two common attribute values must include a *SET*. (For example, consider only the 9 red-open cards, and prove that every group of 5 cards must contain a *SET*.)
2. Prove that 10 cards that have one common attribute value must include a *SET*. (For example, prove that 10 red cards must contain a *SET*.)
3. If 12 randomly selected cards don't contain a *SET* and 3 additional cards are added, what is the probability of a *SET* being present?
4. What is the probability that the game will end with 0, 3, 6, 9, & l2 cards?
5. What is the probability of having 2 disjoint *SETs* among 12 randomly selected cards?
6. Find the maximum number of cards that contain no *SETs*. Prove that you have a maximum.
7. How does the game change, and how do the answers to some of these questions change if you combine 2 or 3 decks of cards together?
8. Prove that among 7 cards there cannot be exactly 4 *SETs*.

Table 4:
More challenging questions

Summary of Pedagogical Concerns: The questions discussed were motivated by thinking about the mathematical results of playing the game of SET®. We have seen how junior high, high school, and college students have also asked some of these same questions and some of their own after playing the game. Students had no trouble with questions #1-3. Working in groups with the use of actual cards and with the occasional prompting from the teacher, students with no background in combinatorics were able to make progress on each of the questions. Those currently in a probability class were exposed to problems that required more creative thinking than many problems in their textbook. Students that developed certain strategies while playing were successful in proving why sometimes no *SETs* existed or explaining why certain types of *SETs* were more common than other types.

Many of the students made connections among the answers to our questions and connections between these questions and ideas from other math or science courses (such as geometry, calculus, and chemistry). At times the college students took longer to solve the problems than the younger students. However, this was because they carefully considered overcounting issues, and they came up with some of our later questions while trying to answer the earlier ones.

Conclusion: The game of SET® provides a multitude of mathematical questions for students of all levels, and students learned a lot of combinatoric theory while having fun. Students developed mathematically sound strategies in order to improve their game. The cards served as manipulatives that students used in order to develop more abstract thinking. Our students enjoyed playing this game, thinking about these questions, and asking their own questions. The game provided an excellent context in which to promote problem solving and deductive reasoning in discrete mathematics, ideas that need to be emphasized in the high school curriculum (NCTM, 1989).

References

Falco, Marsha. (1988). **SET® : The Family Game of Visual Perception.** Fountain Hills, AZ: Set Enterprises, Inc.

National Council of Teachers of Mathematics. (1989). **Curriculum and evaluation standards for school mathematics** . Reston, VA: NCTM.

Miller, Gene. "The Recreational Puzzle Archives on SET®. "http://www.knowitall.com/people/mag/Set/SetRPArchive.txt"(April 28, 1997).

SET Enterprises, Inc. (1998). "The SET® Game Company Homepage. http://www.setgame.com/(April 30, 1998).

Endnotes

1 The SET® game idea and graphics are copyrighted property of SET Enterprises, Inc. and are used for educational purposes by the authors with the written permission (April 13, 1997) of Bob Falco of SET Enterprises, Inc. The SET® name and logo are registered trademarks of SET Enterprises, Inc.

2 Among interested players on the Internet, the maximum number of cards that contain no set is widely believed to be 20. We searched through relevant journals and on the Internet for a proof. If it has already been proven, this fact has not been widely reported.

REFERENCES

- **SET Daily Puzzle**
  To play the FREE daily SET Puzzle, please visit our website at http://www.setgame.com/set/daily_puzzle.

- **SET Daily Puzzle on The New York Times**
  Play 4 FREE daily SET Puzzles at www.nytimes.com/set. This site has 2 basic puzzles and 2 advanced puzzles every day.

- **SET Tutorial**
  The SET interactive Tutorial is available on our website at www.setgame.com/set_tutorial. In the SET tutorial, you'll meet your interactive guide, "Guy". Guy is there to walk you through how to play SET and show you how to make *SETs*.

- **Minds on Math 9**
  R. Alexander et al, Addison Wesley Publishers, Ltd. Don Mills, Ontario, Canada, 1994, Chapter 8 *Polynomials.* This text book has a wonderfully interesting way to use the idea of the SET® Game to teach polynomials.